Algebra
Applied Mathematics
Calculus and Analysis
Discrete Mathematics
Foundations of Mathematics
Geometry
History and Terminology
Number Theory
Probability and Statistics
Recreational Mathematics
Topology

Alphabetical Index
Interactive Entries
Random Entry
New in *MathWorld*

MathWorld Classroom

About *MathWorld*
Send a Message to the Team

Order book from Amazon

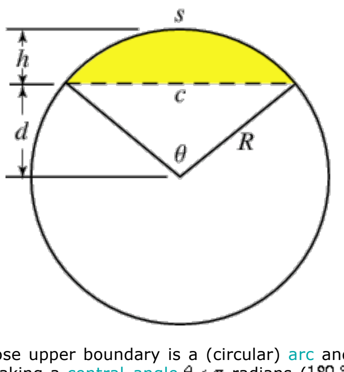
Last updated:
12,704 entries
Sun Aug 26 2007

Created, developed, and nurtured by Eric Weisstein at Wolfram Research

Geometry > Plane Geometry > Arcs

Circular Segment

COMMENT On this Page DOWNLOAD *Mathematica* Notebook



A portion of a disk whose upper boundary is a (circular) arc and whose lower boundary is a chord making a central angle $\theta < \pi$ radians ($180°$), illustrated above as the shaded region. The entire wedge-shaped area is known as a circular sector.

Let R be the radius of the circle, c the chord length, s the arc length, h the height of the arced portion, and d the height of the triangular portion. Then the radius is

$$R = h + d, \tag{1}$$

the arc length is

$$s = R\theta, \tag{2}$$

the height d is

$$d = R\cos\left(\tfrac{1}{2}\theta\right) \tag{3}$$
$$= \tfrac{1}{2}c\cot\left(\tfrac{1}{2}\theta\right) \tag{4}$$
$$= \tfrac{1}{2}\sqrt{4R^2 - c^2}, \tag{5}$$

and the length of the chord is

$$c = 2R\sin\left(\tfrac{1}{2}\theta\right) \tag{6}$$
$$= 2d\tan\left(\tfrac{1}{2}\theta\right) \tag{7}$$
$$= 2\sqrt{R^2 - d^2} \tag{8}$$
$$= 2\sqrt{h(2R - h)}. \tag{9}$$

From elementary trigonometry, the angle θ obeys the relationships

$$\theta = \frac{s}{R} \tag{10}$$
$$= 2\cos^{-1}\left(\frac{d}{R}\right) \tag{11}$$
$$= 2\tan^{-1}\left(\frac{c}{2d}\right) \tag{12}$$
$$= 2\sin^{-1}\left(\frac{c}{2R}\right). \tag{13}$$

The area A of the (shaded) segment is then simply given by the area of the circular sector (the entire wedge-shaped portion) minus the area of the bottom triangular portion,

$$A = A_{\text{sector}} - A_{\text{isosceles triangle}}. \tag{14}$$

Plugging in gives

$$A = \tfrac{1}{2}R^2(\theta - \sin\theta) \tag{15}$$
$$= \tfrac{1}{2}(Rs - cd) \tag{16}$$
$$= R^2\cos^{-1}\left(\frac{d}{R}\right) - d\sqrt{R^2 - d^2} \tag{17}$$
$$= R^2\cos^{-1}\left(\frac{R-h}{R}\right) - (R-h)\sqrt{2Rh - h^2}, \tag{18}$$

where the formula for the isosceles triangle in terms of the polygon vertex angle has been used (Beyer 1987). These formula find application in the common case of determining the volume of fluid in a cylindrical segment (i.e., horizontal cylindrical tank) based on the height of the fluid in the tank.

The area can also be found directly by integration as

$$A = \int_{-R\sin\left(\frac{1}{2}\theta\right)}^{R\sin\left(\frac{1}{2}\theta\right)}\int_{R\cos\left(\frac{1}{2}\theta\right)}^{\sqrt{R^2-x^2}} dy\,dx. \tag{19}$$

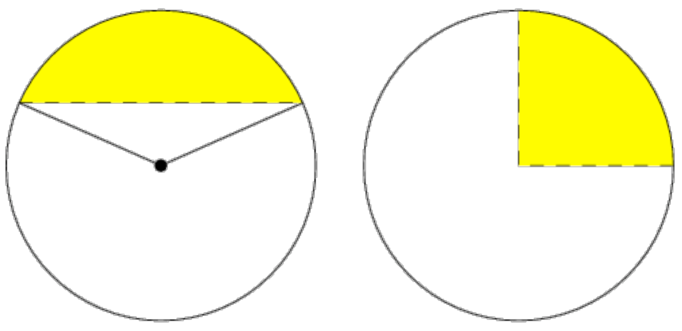
It follows that the weighted mean of y is

$$\langle y\rangle = \int_{-R\sin\left(\frac{1}{2}\theta\right)}^{R\sin\left(\frac{1}{2}\theta\right)}\int_{R\cos\left(\frac{1}{2}\theta\right)}^{\sqrt{R^2-x^2}} y\,dy\,dx \tag{20}$$
$$= \tfrac{2}{3}R^3\sin^3\left(\tfrac{1}{2}\theta\right), \tag{21}$$

so the geometric centroid of the circular segment is

$$\bar{y} = \frac{\langle y\rangle}{A} = \frac{4R\sin^3\left(\frac{1}{2}\theta\right)}{3(\theta - \sin\theta)}. \tag{22}$$

Checking shows that this obeys the proper limits $\bar{y} = 4R/(3\pi)$ for a semicircle ($\theta = \pi$) and $\bar{y} = R$ for a point mass at the top of the segment ($\theta \to 0$).



To find the value of h such that the circular segment has area equal to 1/4 that of the circle, plug $A = \pi R^2/4$ into equation (22) and divide both sides by R^2 to get

$$\tfrac{1}{4}\pi = \cos^{-1}(1-x) - (1-x)\sqrt{2x - x^2}, \tag{23}$$

where $x \equiv h/R$. This cannot be solved analytically, but the solution can be found numerically to be approximately $x = 0.596027$, corresponding to $h = 0.596027\,R$.

Approximate formulas for the arc length and area are

$$s \approx \sqrt{c^2 + \tfrac{16}{3}h^2}, \tag{24}$$

accurate to within 0.3% for $0° \le \theta \le 90°$, and

$$A \approx \tfrac{2}{3}ch + \frac{h^3}{2c}, \tag{25}$$

accurate to within 0.1% for $0° \le \theta \le 150°$ and 0.8% for $150° \le \theta \le 180°$ (Harris and Stocker 1998).

SEE ALSO: Chord, Circle-Circle Intersection, Circular Sector, Cylindrical Segment, Lens, Parabolic Segment, Reuleaux Triangle, Sagitta, Spherical Segment.
[Pages Linking Here]

REFERENCES:

Beyer, W. H. (Ed.). *CRC Standard Mathematical Tables, 28th ed.* Boca Raton, FL: CRC Press, p. 125, 1987.

UPDATE THIS LINK Car Talk @ cars.com. "The Great Gas Tank Math Solved." http://cartalk.cars.com/Radio/call/dipstick/proof.html.

Fukagawa, H. and Pedoe, D. "Segments of a Circle." §1.6 in *Japanese Temple Geometry Problems.* Winnipeg, Manitoba, Canada: Charles Babbage Research Foundation, pp. 14-15 and 88-92, 1989.

Harris, J. W. and Stocker, H. "Segment of a Circle." §3.8.6 in *Handbook of Mathematics and Computational Science.* New York: Springer-Verlag, pp. 92-93, 1998.

Kern, W. F. and Bland, J. R. *Solid Mensuration with Proofs, 2nd ed.* New York: Wiley, p. 4, 1948.

LAST MODIFIED: April 9, 2003

CITE THIS AS:

Weisstein, Eric W. "Circular Segment." From *MathWorld*--A Wolfram Web Resource. http://mathworld.wolfram.com/CircularSegment.html